JOURNAL OF GUIDANCE, CONTROL, AND DYNAMICS
Vol. 21, No. 3, May–June 1998

Closed-Loop Identification of Flexible Structures: An Experimental Example

Roy S. Smith*
University of California, Santa Barbara, Santa Barbara, California 93106

The application of a closed-loop identification procedure to a flexible structure, the Jet Propulsion Laboratory Control Structure Interaction Phase B testbed, is described. The approach is based on an indirect, closed-loop identification procedure, recently developed by Van den Hof and Schrama (Van den Hof, P. M., and Schrama, R. J., "An Indirect Method for Transfer Function Estimation from Closed-Loop Data," *Automatica*, Vol. 29, No. 6, 1993, pp. 1523–1527), which gives a consistent model estimate in the case where the system noise/output disturbance model is not accurate. The procedure is modified and applied, in the frequency domain, to closed-loop data from a flexible structure control experiment. The results of the analysis are compared to a purely open-loop frequency-domain identification approach. Enhancements to the experimental configuration are also discussed.

I. Introduction

FEEDBACK control for active vibration suppression is an area of significant importance to large spaceborne structures. The majority of control design techniques applied to this problem require a mathematical model of the system, compatible with the proposed design methodology. The identification of such models has been an area of active research interest within the flexible structure control design community. This paper examines a closed-loop approach to this problem, using frequency-domain data.

Several identification approaches have received widespread attention, and for the most part, these are open-loop techniques. The eigensystem realization algorithm (ERA)[1] has successfully been applied to the identification of flexible structures.[2,3] This algorithm also forms the basis of a number of related approaches (see, for example, Refs. 4–8). The ERA approach has also been combined with minimum model error methods.[9,10] These methods use impulse response or free-decay data and, with a sufficiently rich excitation signal, can also be applied to frequency-domain data. A more direct time-domain approach is used by the observer/Kalman filter identification (OKID) algorithms (see Refs. 11 and 12 for further details). Much of the ERA-based work is detailed by Juang.[13]

The q-Markov cover approach has also been successfully applied to flexible structure systems (see Refs. 14–16 for algorithmic details and Refs. 17 and 18 for application to the Mini–Mast and Hubble space telescope problems). The ERA and q-Markov cover approaches are compared on a simulated flexible structure problem in Ref. 19. Other open-loop methods are discussed in Refs. 20–22. This research area is of considerable importance and the preceding discussion is by no means exhaustive.

There has been an increase in research interest in closed-loop identification of models for control design purposes. There are several reasons for this. In some cases a controller is required to actually carry out the experiments. Open-loop unstable systems are one example, although these do not typically occur in flexible structure vibration suppression problems.

There has been recent work on the iterative application of identification and control design methods. These approaches have attempted to systematically use the controller to obtain additional, control-relevant information about the system. The ability to identify a system in closed-loop operation is an integral part of any such scheme. The reader is referred to Refs. 23–25 and the references therein for additional details. It is also possible that identification under closed-loop operation gives a model that is more relevant for control design purposes than one obtained via open-loop methods. This issue is addressed further in the discussion section of this paper.

Closed-loop identification has also attracted interest within the flexible structure research community. Previous work has looked at adaptive identification (for example, Ref. 26) and at approaches involving the use of experiment design based on previous data.[5,27,28] The work described in Refs. 29–31 studies iterative identification and design methods in the frequency domain. The OKID approach has recently been extended to the closed-loop case in Refs. 32 and 33. The point is made in this work[32,33] that the characteristics of the noise strongly affect the result.

The approach taken here is motivated by that of Van den Hof and Schrama.[34] This is an indirect, two-step method that alleviates the noise sensitivity problems mentioned earlier. Their work[34] is briefly summarized here. The principles of this method are applied to closed-loop frequency domain data from the Jet Propulsion Laboratory (JPL) Control Structure Interaction (CSI) Phase B flexible structure. Comparison is made between the results of the open-loop and closed-loop identification techniques.

The paper is arranged as follows. An outline of the theoretical identification procedure is given in Sec. II. This is applied to the JPL CSI Phase B structural identification problem in Sec. III. The configuration used was such that a mixture of open- and closed-loop data was required to obtain estimated transfer functions. Section IV presents the results and compares them to the purely open-loop identification procedures. Section V discusses the configuration and experiments required to obtain a system model from purely closed-loop experiments. A brief summary and speculations on the potential benefits of closed-loop identification are given in Sec. VI.

II. Closed-Loop Identification Approach

Van den Hof and Schrama[34] consider the identification of the plant G_0 in the closed-loop configuration shown in Fig. 1. The signal $n = H_0(z^{-1})e$ is the noise process, with the usual assumptions; $H_0(z^{-1})$ is stable and stably invertible, and e is a zero mean unit variance white noise signal.

They[34] make the point that the output is given by

$$y = G_0(z^{-1})u + H_0(z^{-1})e \tag{1}$$

but any open-loop technique used to estimate $G_0(z^{-1})$ will give a noise-dependent bias error as u is not uncorrelated with e. The consequences of applying open-loop methods to closed-loop data are discussed in greater detail at the end of this section.

By defining the sensitivity function,

$$S_0(z^{-1}) = \frac{1}{1 + G_0(z^{-1})C(z^{-1})} \tag{2}$$

Received Dec. 22, 1994; revision received Oct. 27, 1997; accepted for publication Nov. 19, 1997.

*Associate Professor, Department of Electrical and Computer Engineering. E-mail: roy@ece.ucsb.edu. Member AIAA.

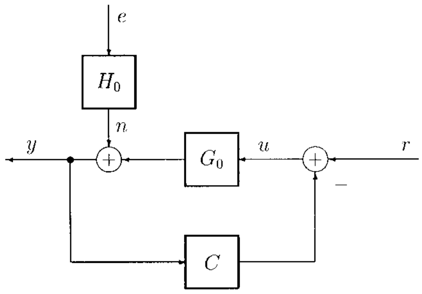

Fig. 1 Closed-loop system for identification: configuration of Van den Hof and Schrama.[34]

the system equations can be rewritten as

$$u = S_0(z^{-1})r - C(z^{-1})S_0(z^{-1})H_0(z^{-1})e \tag{3}$$

and

$$y = G_0(z^{-1})S_0(z^{-1})r + H_0(z^{-1})e \tag{4}$$

Note that r and e are uncorrelated and $S_0(z^{-1})$ can, therefore, be estimated by measuring u and r. Standard open-loop identification techniques can be applied to this problem (see, for example, Ljung[35]). Now define the fictitious signal

$$u^r := S_0(z^{-1})r \tag{5}$$

and note that

$$y = G_0(z^{-1})u^r + S_0(z^{-1})H_0(z^{-1})e \tag{6}$$

Clearly, u^r is also independent of e and this equation can be used to estimate $G_0(z^{-1})$. In practice, only an estimate of u^r can be obtained. The method consists of a two-step procedure.

1) Estimate $S_0(z^{-1})$ from r and u and calculate an estimate of u^r.

2) Estimate $G_0(z^{-1})$ from y and the estimate of u^r.

Van den Hof and Schrama[34] show that if $S_0(z^{-1})$ can be estimated accurately, then $G_0(z^{-1})$ can also be estimated accurately. There is no requirement for exact (or consistent) estimation of $S_0(z^{-1})$. More precisely, they[34] employ a prediction error approach where a noise model, $H(z^{-1})$ is specified. It is not necessary that the model $H(z^{-1})$ closely approximate the true noise filter $H_0(z^{-1})$ and, as is typically done in the open-loop case, $H(z^{-1})$ can include frequency weighting prefilters. The plant model is parameterized by $G(z^{-1}, \theta)$ and the sensitivity function is parameterized by $S(z^{-1}, \beta)$. A fixed noise model, including any prefiltering and denoted by $R(z^{-1})$, is assumed for the identification of $S(z^{-1})$.

Under weak conditions (as specified by Ljung,[35] p. 210), as the length of the time record approaches infinity, a prediction error scheme will give $\theta \to \theta^*$, where

$$\theta^* = \arg\min_\theta \int_{-\pi}^{\pi} \Big|G_0(e^{j\omega})S_0(e^{j\omega}) - G(e^{j\omega}, \theta)S(e^{j\omega}, \beta^*)\Big|^2 \frac{\psi_r(\omega)}{|H(e^{j\omega})|^2}\, \mathrm{d}\omega \tag{7}$$

Here $\psi_r(\omega)$ is the spectrum of r and β^* arises from the estimation of $S_0(z^{-1})$. More precisely,

$$\beta^* = \arg\min_\beta \int_{-\pi}^{\pi} \Big|S_0(e^{j\omega}) - S(e^{j\omega}, \beta)\Big|^2 \frac{\psi_r(\omega)}{|R(e^{j\omega})|^2}\, \mathrm{d}\omega \tag{8}$$

The effect of the estimation of $S_0(z^{-1})$ on the final result can be seen by noting that

$$\begin{aligned} &G_0(e^{j\omega})S_0(e^{j\omega}) - G(e^{j\omega}, \theta)S(e^{j\omega}, \beta^*) \\ &= \big[G_0(e^{j\omega}) - G(e^{j\omega}, \theta)\big]S_0(e^{j\omega}) \\ &+ G(e^{j\omega}, \theta)\big[S_0(e^{j\omega}) - S(e^{j\omega}, \beta^*)\big] \end{aligned} \tag{9}$$

Therefore, if the error in estimating $S_0(e^{j\omega})$ is small, it will have limited effect on the error in estimating $G_0(e^{j\omega})$. Note that there is no penalty in estimating $S_0(z^{-1})$ with a very high-order model; it is only used to calculate u^r and does not affect the order of the $G_0(z^{-1})$ model.

It is interesting to compare the preceding two-stage method to the more common approach of using a prediction error scheme directly on the closed-loop data. In the open-loop case ($C = 0$) with a fixed noise model, $H(e^{j\omega})$, a prediction error scheme will give $\theta \to \theta^*$, where

$$\theta^* = \arg\min_\theta \int_{-\pi}^{\pi} \Big[\big|G_0(e^{j\omega}) - G(e^{j\omega}, \theta)\big|^2 \psi_u(\omega)\Big] \frac{1}{|H(e^{j\omega})|^2}\, \mathrm{d}\omega \tag{10}$$

where $\psi_u(\omega)$ is the spectrum of the input u.

If we now apply this approach with closed-loop data the estimate will converge to

$$\theta^* = \arg\min_\theta \int_{-\pi}^{\pi} \epsilon(\omega)\, \mathrm{d}\omega \tag{11}$$

where

$$\epsilon(\omega) = \left\{\Big|S_0(e^{j\omega})\big[G_0(e^{j\omega}) - G(e^{j\omega}, \theta)\big]\Big|^2 \psi_r(\omega) + \frac{\big|S_0(e^{j\omega})\big|^2}{|S(e^{j\omega}, \theta)|^2}\psi_n(\omega)\right\} \frac{1}{|H(e^{j\omega})|^2}$$

and $\psi_n(\omega) = |H_0(e^{j\omega})|^2$ is the spectrum of the noise signal, n. Both the Van den Hof and Schrama[34] two-stage method [see Eq. (7)] and the direct prediction error method with closed-loop data [see Eq. (11)] weight the plant model fit $[G(e^{j\omega}, \theta) - G_0(e^{j\omega})]$ by the actual closed-loop sensitivity function. This is a consequence of using closed-loop data rather than the particular method chosen.

Note, however, that in the direction prediction error approach [Eq. (11)] θ^* depends on the spectrum of the noise, $\psi_n(\omega)$, resulting in a noise-dependent bias error. Ljung[36] points out that consistent closed-loop estimates can be obtained via prediction error methods if a noise model is also consistently identified. This is not a requirement in the Van den Hof and Schrama[34] approach.

III. Application to the Flexible Structure

The approach of the preceding section motivates the following application to the structure. There are several differences, which arise mostly from the form of the data. The work given here will be concerned only with obtaining smoothed transfer function estimates in the frequency domain. The subsequent stage of fitting state-space realizations to the frequency-domain data is discussed elsewhere.[21]

The experimental system is the JPL Phase B structure, which was designed as an experimental testbed for studying the control–structure interaction problems that are expected to arise in large spaceborne optical telescopes. The structure is shown schematically in Fig. 2. It is approximately 8 ft tall and supports a moving trolley, which is used for optical pathlength compensation. Hardware details and initial modeling and identification results can be found in Refs. 37 and 38. The problem considered here is vibration suppression in the truss structure itself. The control is actuated via an active member (AM) in the lower corner of the truss. Control measurements were taken from X and Y direction accelerometers mounted on the trolley and collocated AM load and AM displacement sensors. Disturbances were introduced into the system via a shaker mounted at the top of the third truss bay. For identification purposes, the input to the shaker was also measured. The details of the open-loop identification procedure and controller design are given in Ref. 39. The control design issues are beyond the scope of this work and will not be discussed further here. Comparison will be made with some of the open-loop identification results.

The theoretical results given in the preceding section consider only a single-input/single-output (SISO) system with time-domain data. The system under consideration here has two inputs and four outputs. Furthermore, the control loop can only be closed around one of the inputs (AM). The closed-loop configuration is shown in Fig. 3. The output $\boldsymbol{y}$ consists of the four measurements: X and Y accelerations, AM load, and AM displacement. The measurement noise is $\boldsymbol{n}$ and the shaker input is v. The actual controller (K_2 in

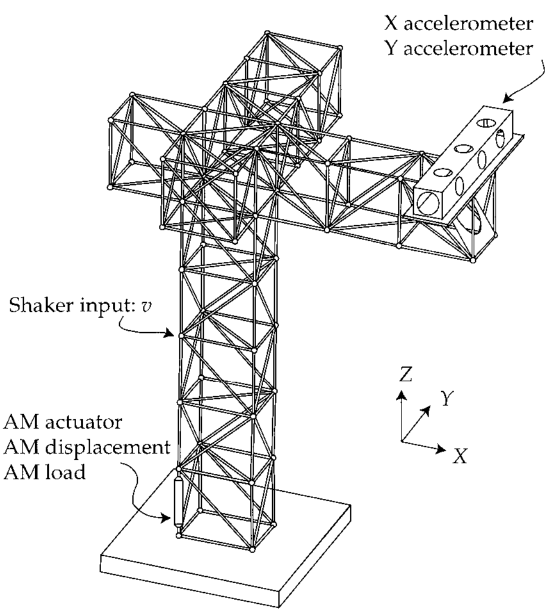

Fig. 2 Scheme of the JPL Phase B structure; AM denotes the active member using for actuation and the collocated part of the sensing.

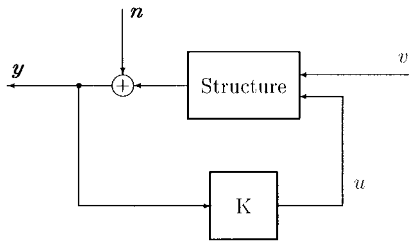

Fig. 3 Configuration for closed loop experiments on the JPL CSI Phase B testbed.

Ref. 39) used affects the quality of the result but does not change the approach in general.

A Tektronix spectrum analyzer was used as the data collection facility. This performed real-time averaging of the data to provide an estimated transfer function at a grid of frequency points. Five averages were used to give 1601 points of data over the range 15–100 rad/s. In the experiment considered here, the analyzer provided the excitation to the shaker, v, and measured the four channels of output data, $\boldsymbol{y}$.

The equations of the system model, in terms of the open-loop components, are given by

$$\boldsymbol{y} = \boldsymbol{P}_{yv} v + \boldsymbol{P}_{yu} u + \boldsymbol{n} \tag{12}$$

$$u = \boldsymbol{K}\boldsymbol{y} \tag{13}$$

where $\boldsymbol{y}$ is a 4×1 vector consisting of the X and Y accelerations and the active member load and displacement measurements. The objective will be to obtain a frequency-domain estimate of $\boldsymbol{P}_{yu}$ (the open-loop transfer function from u to $\boldsymbol{y}$) from closed-loop data.

The available data come from two experiments. The first involves estimating the closed-loop transfer function from v to $\boldsymbol{y}$: $\boldsymbol{T}_{yv}$. This estimate, $\tilde{\boldsymbol{T}}_{yv}$, comes directly from the spectrum analyzer. The second experiment provides a closed-loop estimate of the transfer function from u to $\boldsymbol{y}$: $\boldsymbol{T}_{yu}$. The estimate is $\tilde{\boldsymbol{T}}_{yu}$. In this case the spectrum analyzer again provided the excitation v to the shaker.

Note that, in the first experiment,

$$\boldsymbol{y} = (I - \boldsymbol{P}_{yu}\boldsymbol{K})^{-1}\boldsymbol{P}_{yv} v + (\boldsymbol{I} - \boldsymbol{P}_{yu}\boldsymbol{K})^{-1}\boldsymbol{n} \tag{14}$$

$$\boldsymbol{y} = \boldsymbol{T}_{yv} v + (\boldsymbol{I} - \boldsymbol{P}_{yu}\boldsymbol{K})^{-1}\boldsymbol{n} \tag{15}$$

and as v and $\boldsymbol{n}$ are uncorrelated, the spectrum analyzer gives an estimate of $\boldsymbol{T}_{yv}$.

In general, P will be used for an open-loop model and T will be used for a closed-loop model. Note that $\boldsymbol{T}_{yv}$ and $\boldsymbol{T}_{yu}$ (and their respective estimates) bear little resemblance to the open-loop transfer functions $\boldsymbol{P}_{yv}$ and $\boldsymbol{P}_{yu}$. This is because, as a result of the control action, the inputs u and v are highly correlated and the measurement noise is now also correlated with u.

To use the Van den Hof and Schrama[34] approach we must use these data to estimate the appropriate sensitivity function. It will be seen that to do this we require an experimental estimate of T_{uv}, the closed-loop transfer function between v and u. Manipulation of Eqs. (12) and (13) shows that

$$T_{uv} = \boldsymbol{K}(\boldsymbol{I} - \boldsymbol{P}_{yu}\boldsymbol{K})^{-1}\boldsymbol{P}_{yv} \tag{16}$$

and

$$u = T_{uv} v + \boldsymbol{K}(\boldsymbol{I} - \boldsymbol{P}_{yu}\boldsymbol{K})^{-1}\boldsymbol{n} \tag{17}$$

Again, v and $\boldsymbol{n}$ are uncorrelated so this can be obtained from a spectrum analyzer by measurement of u and v. The required data are not available in this case, so an alternative approach is used.

Substituting Eq. (17) into Eq. (12) and comparing with Eq. (15) shows that

$$\boldsymbol{T}_{yv} = \boldsymbol{P}_{yv} + \boldsymbol{P}_{yu} T_{uv} \tag{18}$$

A similar approach is used to determine $\boldsymbol{T}_{yu}$. To proceed we must assume that T_{uv} is invertible, at least on a frequency by frequency basis. If T_{uv} is not invertible it means that there are signals at v that cause no control action. Either these signals do not appear at $\boldsymbol{y}$, or there exist signals at $\boldsymbol{y}$ that cause no controller action. For any strictly proper controller, this condition is, strictly speaking, violated at infinite frequency. It will be seen that this condition is only required over a finite frequency range.

Assuming that T_{uv}^{-1} exists, multiplication of Eq. (17) by T_{uv}^{-1} leads to the following expression for v:

$$v = T_{uv}^{-1} u - T_{uv}^{-1}\boldsymbol{K}(\boldsymbol{I} - \boldsymbol{P}_{yu}\boldsymbol{K})^{-1}\boldsymbol{n} \tag{19}$$

Substituting this into Eq. (12) gives

$$\boldsymbol{y} = \left(\boldsymbol{P}_{yu} + \boldsymbol{P}_{yv} T_{uv}^{-1}\right)u + \left(\boldsymbol{I} - T_{yv} T_{uv}^{-1}\right)(\boldsymbol{I} - \boldsymbol{P}_{yu}\boldsymbol{K})^{-1}\boldsymbol{n} \tag{20}$$

Clearly, then,

$$\boldsymbol{T}_{yu} = \boldsymbol{P}_{yu} + \boldsymbol{P}_{yv} T_{uv}^{-1} \tag{21}$$

Combining Eqs. (21) and (18) leads to

$$\boldsymbol{T}_{yu} T_{uv} = \boldsymbol{T}_{yv} \tag{22}$$

This can now be used to obtain an experimentally based estimate of T_{uv}.

Both $\boldsymbol{T}_{yu}$ and $\boldsymbol{T}_{yv}$ are 4×1 column vectors, for which experimental estimates $\tilde{\boldsymbol{T}}_{yu}$ and $\tilde{\boldsymbol{T}}_{yv}$ are available. We can now obtain an estimate of T_{uv} by solving

$$\tilde{\boldsymbol{T}}_{yu} T_{uv} = \tilde{\boldsymbol{T}}_{yv} \tag{23}$$

T_{uv} is a SISO system and is, in general, overdetermined by this equation. A least squares fit was used to estimate T_{uv}. This estimate is $\tilde{T}_{uv}$ and, for the experiment outlined in the subsequent section, is shown in Fig. 4. Also shown are the four element by element estimates. These are almost identical in the frequency range of the lower modes of interest (30–80 rad/s).

The open-loop transfer functions of interest, $\boldsymbol{P}_{yu}$ and $\boldsymbol{P}_{yv}$, are related to the closed-loop transfer functions $\boldsymbol{T}_{yu}$ and $\boldsymbol{T}_{yv}$ by Eqs. (21) and (18). Even with an estimate of T_{uv}, there is insufficient information to solve for $\boldsymbol{P}_{yu}$ and $\boldsymbol{P}_{yv}$.

To alleviate this problem we will use an open-loop estimate for $\boldsymbol{P}_{yv}$, denoted by $\hat{\boldsymbol{P}}_{yv}$, and solve for $\boldsymbol{P}_{yu}$ via Eq. (21). Denote the resulting estimate of $\boldsymbol{P}_{yu}$ by $\tilde{\boldsymbol{P}}_{yu}$, which is given by finding a solution to

$$\tilde{\boldsymbol{P}}_{yu} = \tilde{\boldsymbol{T}}_{yu} - \hat{\boldsymbol{P}}_{yv} \tilde{T}_{uv}^{-1} \tag{24}$$

Note that $\boldsymbol{P}_{yv}$ could also have been estimated from the closed-loop data and an open-loop estimate of $\boldsymbol{P}_{yu}$.

The control loop is closed around $\boldsymbol{P}_{yu}$, making the closed-loop estimate of $\boldsymbol{P}_{yu}$ more appropriate to the control design problem. The experimental results of the next section use Eq. (24) to estimate $\boldsymbol{P}_{yu}$.

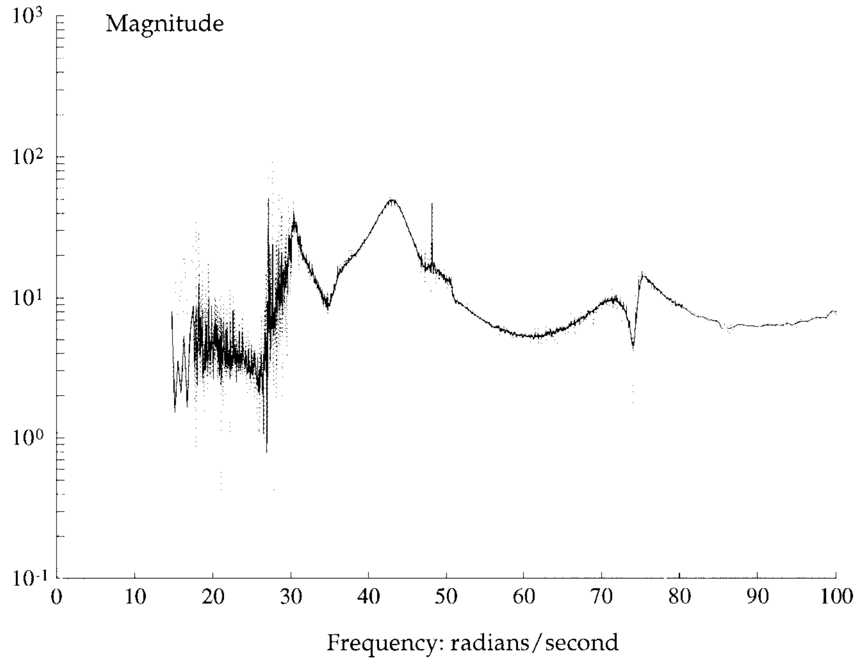

Fig. 4 Element by element estimates of T_{uv} (· · · · ·) and least squares estimate $\tilde{T}_{uv}$ (——).

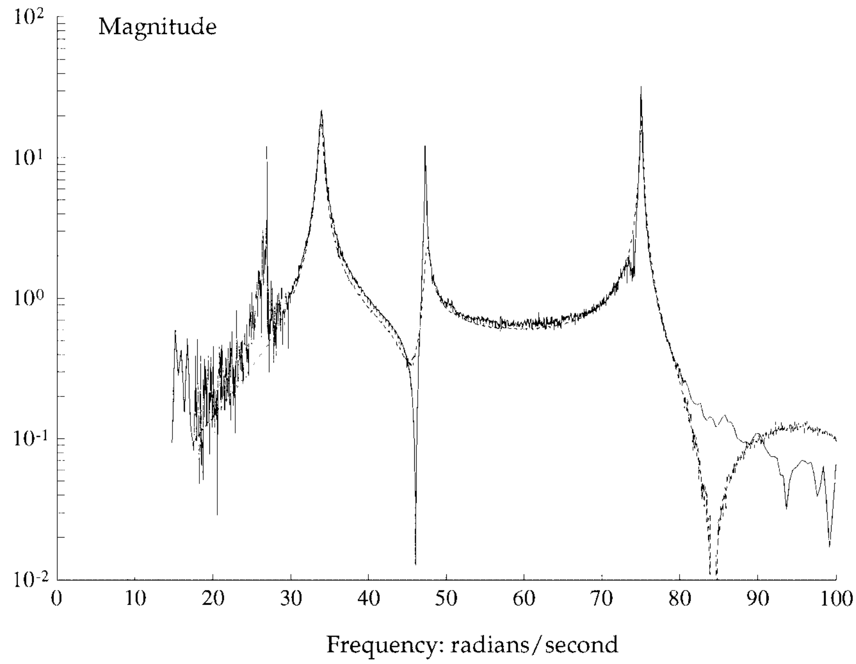

Fig. 5 Estimates of P_{yu} (Y direction): closed loop, $\tilde{P}_{yu}$ (——), and open loop, $\hat{P}_{yu}$ (- - -).

IV. Experimental Results

Experimental frequency-domain transfer function estimates $\tilde{\boldsymbol{T}}_{yu}$ and $\tilde{\boldsymbol{T}}_{yv}$ were taken for the Phase B structure operating in closed loop. The described procedure was used to obtain the estimate $\tilde{T}_{uv}$ (shown in Fig. 4). The estimate of $\boldsymbol{P}_{yu}$ was obtained from an open-loop estimate of $\boldsymbol{P}_{yv}$ and Eq. (24). The results are shown in Fig. 5. The closed-loop estimate $\tilde{\boldsymbol{P}}_{yu}$ is compared to an open-loop estimate $\hat{\boldsymbol{P}}_{yu}$. Only the Y direction is illustrated for clarity. The X direction estimate is qualitatively similar. The AM load and AM displacement estimates show significantly greater agreement between the open- and closed-loop estimates.

The first and third modes (34 and 75 rad/s) show very good agreement between the open- and closed-loop experiments. The second mode (47 rad/s) shows significantly less damping in the closed-loop estimate. A similar effect was observed in other closed-loop experiments; the closed-loop second mode damping was less than predicted by the open-loop design models. This suggests that the closed-loop estimate, given here, is a better predictor of closed-loop performance. This issue is discussed further in the Conclusions section.

The closed-loop experiment estimate $\tilde{\boldsymbol{P}}_{yu}$ differs significantly in the 80–100 rad/s region. This is more likely due to the effect of noise on the signals used in the calculation than a different behavior in closed loop. Note that the quality of the estimate $\tilde{\boldsymbol{P}}_{yu}$ depends on the quality of the original open-loop estimate of $\boldsymbol{P}_{yv}$. The noisy mode appearing at approximately 30 rad/s is due to a local mode in the shaker. This is probably an artifact that could be removed with the improved experimental technique suggested next.

V. Augmented Configuration

The preceding experimental analysis used a hybrid open- and closed-loop approach. We now outline the experimental configuration changes required to estimate all transfer functions from closed-loop data.

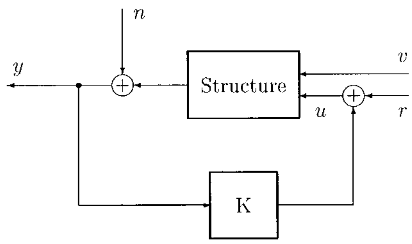

Fig. 6 Modified closed-loop identification configuration for the JPL CSI Phase B testbed.

Consider the inclusion of an additional input r, as shown in Fig. 6. The inputs v and r are generated for the identification experiments and are required to be uncorrelated.

In this case the system equations are given by

$$\boldsymbol{y} = \boldsymbol{P}_{yu} u + \boldsymbol{P}_{yv} v + \boldsymbol{n} \tag{25}$$

$$u = r + \boldsymbol{K y} \tag{26}$$

Rearranging Eqs. (25) and (26) gives

$$u = T_{ur} r + T_{uv} v + (I - \boldsymbol{K P}_{yu})^{-1} \boldsymbol{K n} \tag{27}$$

where

$$T_{ur} := (I - \boldsymbol{K P}_{yu})^{-1} \tag{28}$$

and

$$T_{uv} := (I - \boldsymbol{K P}_{yu})^{-1} \boldsymbol{K P}_{yv} \tag{29}$$

As r, v, and $\boldsymbol{n}$ are uncorrelated, an estimate of T_{ur}, $(\tilde{T}_{ur})$, can be obtained from closed-loop experimental data.

Substituting Eq. (27) into Eq. (25) and rearranging leads to

$$\boldsymbol{y} = (\boldsymbol{P}_{yv} + \boldsymbol{P}_{yu} T_{uv}) v + \boldsymbol{P}_{yu} T_{ur} r + (\boldsymbol{I} + \boldsymbol{P}_{yu} \boldsymbol{K})^{-1} \boldsymbol{n} \tag{30}$$

Define $\boldsymbol{T}_{yr}$ by

$$\boldsymbol{T}_{yr} = \boldsymbol{P}_{yu} T_{ur} \tag{31}$$

and note that an estimate of $\boldsymbol{T}_{yr}$, $(\tilde{\boldsymbol{T}}_{yr})$, can also be obtained directly from the closed-loop experimental data. Now Eq. (31) can be used as the basis of the approach to estimate $\boldsymbol{P}_{yu}$. To do this, use the estimate $\tilde{\boldsymbol{P}}_{yu}$ that minimizes the error in the equation,

$$\tilde{\boldsymbol{T}}_{yr} = \tilde{\boldsymbol{P}}_{yu} \tilde{T}_{ur} \tag{32}$$

Unlike the results presented in the preceding section, there is no need to use any open-loop data or model in the identification procedure. The estimate of T_{ur} plays an analogous role to the estimate of the sensitivity function, $S_0(e^{j\omega})$, in Sec. II.

VI. Conclusions

We have addressed the issue of closed-loop identification of flexible structures in the frequency domain. It has been shown that the effects of the correlation between the noise and the controller output can be effectively removed from the identification problem. A hybrid approach, using an open-loop estimate of a part of the system, has been applied to the closed-loop identification of the JPL CSI Phase B testbed.

This closed-loop estimate of the transfer function showed less damping in one mode than that estimated from purely open-loop data. This phenomenon was also observed in other closed-loop control experiments. This suggests, at least for this example, that the closed-loop experiments were better in terms of giving a model that more correctly predicted closed-loop behavior. There are several possible reasons why this may be the case.

The first is that under reasonable closed-loop operation, the magnitude of the system displacements are much smaller than they would be in the case of an open-loop identification experiment. If amplitude-dependent nonlinearities (with effects that increase with increasing amplitude) exist in the system, then closed-loop operation will exhibit less nonlinear behavior. This will give a more accurate result for identification schemes that yield linear models.

The second issue is that closed-loop operation will automatically bias the system at the desired closed-loop operating point. Again, if the system has operating-point-dependent nonlinearities closed-loop identification will alleviate bias difference errors, which may arise in open-loop identification.

A linear analysis is unable to shed light on these issues as both issues arise from nonlinear or unmodeled effects. This argues for an experimental assessment of the pros and cons of closed-loop identification. Note also that both issues apply to models obtained for the purpose of control system design.

We have also presented an alternative experimental configuration that allows an open-loop model to be obtained entirely from closed-loop experimental data. The experimental results presented suggest that closed-loop experiments can give useful control design models. The experimental assessment of this issue on flexible structures is left for future work. Only a thorough assessment of closed-loop performance, for a variety of controllers, will provide a practical answer to the question of applicability of such procedures.

Acknowledgments

This work was supported by NASA under a contract to the Jet Propulsion Laboratory Control Structure Interaction program. The author would also like to thank Cheng-Chih Chu for assistance in the closed-loop design aspects related to this problem; Dankai Liu and John O'Brien for performing experiments; and John Spanos, James Fanson, and John Garba for useful discussions on the problem.

References

[1]Juang, J.-N., and Pappa, R., "An Eigensystem Realization Algorithm for Modal Parameter Estimation and Model Reduction," *Journal of Guidance, Control, and Dynamics*, Vol. 8, No. 5, 1985, pp. 620–627.

[2]Cooper, J., and Wright, J., "Spacecraft In-Orbit Identification Using Eigensystem Realization Methods," *Journal of Guidance, Control, and Dynamics*, Vol. 15, No. 2, 1992, pp. 352–359.

[3]Kim, H., VanHorn, D., and Doiron, H., "Free-Decay Time-Domain Modal Identification for Large Space Structures," *Journal of Guidance, Control, and Dynamics*, Vol. 17, No. 3, 1994, pp. 513–519.

[4]Longman, R., and Juang, J.-N., "Recursive Form of the Eigensystem Realization Algorithm for System Identification," *Journal of Guidance, Control, and Dynamics*, Vol. 12, No. 5, 1989, pp. 647–652.

[5]Yang, C.-D., and Yeh, F.-B., "Identification, Reduction and Refinement of Model Parameters by the Eigensystem Realization Algorithm," *Journal of Guidance, Control, and Dynamics*, Vol. 13, No. 6, 1990, pp. 1051–1059.

[6]Juang, J.-N., and Phan, M., "Linear System Identification via Backward-Time Observer Models," *Journal of Guidance, Control, and Dynamics*, Vol. 17, No. 3, 1994, pp. 505–512.

[7]Horta, L., Phan, M., Juang, J.-N., Longman, R., and Sulla, J., "Frequency-Weighted System Identification and Linear Quadratic Controller Design," *Journal of Guidance, Control, and Dynamics*, Vol. 16, No. 2, 1993, pp. 330–336.

[8]Quan, R., "System Identification Using Frequency Scanning and the Eigensystem Realization Algorithm," *Journal of Guidance, Control, and Dynamics*, Vol. 17, No. 4, 1994, pp. 670–675.

[9]Roemer, M., and Mook, D., "Robust Modal Identification/Estimation of the Mini-Mast Testbed," *Journal of Guidance, Control, and Dynamics*, Vol. 15, No. 3, 1992, pp. 646, 647.

[10]Mook, D., and Junkins, J., "Minimum Model Error Estimation for Poorly Modeled Dynamic Systems," *Journal of Guidance, Control, and Dynamics*, Vol. 11, No. 3, 1988, pp. 256–261.

[11]Chen, C.-W., Huang, J.-K., Phan, M., and Juang, J.-N., "Integrated System Identification and State Estimation for Control of Flexible Space Structures," *Journal of Guidance, Control, and Dynamics*, Vol. 15, No. 1, 1992, pp. 88–95.

[12]Juang, J.-N., Phan, M., Horta, L. G., and Longman, R., "Identification of Observer/Kalman Filter Markov Parameters: Theory and Experiments," *Journal of Guidance, Control, and Dynamics*, Vol. 16, No. 2, 1993, pp. 320–329.

[13]Juang, J.-N., *Applied System Identification*, Prentice–Hall, Englewood Cliffs, NJ, 1994.

[14]Wagie, D., and Skelton, R., "A Projection Approach to Covariance Equivalent Realizations of Discrete Systems," *IEEE Transactions on Automatic Control*, Vol. 31, No. 12, 1986, p. 1114.

[15]Anderson, B., and Skelton, R., "The Generation of All q-Markov Covers," *IEEE Transactions on Circuits and Systems*, Vol. 35, No. 4, 1988, pp. 375–384.

[16]King, A., Desai, U., and Skelton, R., "A Generalized Approach to q-Markov Covariance Equivalent Realizations of Discrete Systems," *Automatica*, Vol. 24, No. 4, 1988, pp. 507–515.

[17]Liu, K., and Skelton, R., "Q-Markov Covariance Equivalent Realization and Its Application to Flexible Structure Identification," *Journal of Guidance, Control, and Dynamics*, Vol. 16, No. 2, 1993, pp. 308–319.

[18]Liu, K., Skelton, R., and Sharkey, J., "Modeling Hubble Space Telescope Flight Data by q-Markov Cover Identification," *Journal of Guidance, Control, and Dynamics*, Vol. 17, No. 2, 1994, pp. 250–256.

[19]Lew, J.-S., Juang, J.-N., and Longman, R., "Comparison of Several System Identification Methods for Flexible Structures," *Journal of Sound and Vibration*, Vol. 167, No. 3, 1993, pp. 461–480.

[20]Tzes, A., and Yurkovich, S., "A Frequency Domain Identification Scheme for Flexible Structure Control," *Journal of Dynamic Systems, Measurement and Control*, Vol. 112, Sept. 1990, pp. 427–434.

[21]Fanson, J., Chu, C.-C., Lurie, B., and Smith, R., "Damping and Structural Control of the JPL Phase 0 Testbed Structure," *Journal of Intelligent Material Systems and Structures*, Vol. 2, July 1991, pp. 281–300.

[22]Banks, H., Reich, S., and Rosen, I., "An Approximation Theory for the Identification on Nonlinear Distributed Parameter Systems," *SIAM Journal of Control and Optimization*, Vol. 28, No. 3, 1990, pp. 552–569.

[23]Gevers, M., "Connecting Identification and Robust Control: A New Challenge," *Proceedings of International Federation of Automatic Control (IFAC) Symposium on Identification and System Parameter Estimation*, Vol. 1, Pergamon, Oxford, England, UK, 1991, pp. 1–10.

[24]Schrama, R. J. P., "Accurate Identification for Control: The Necessity of an Iterative Scheme," *IEEE Transactions on Automatic Control*, Vol. 37, July 1992, pp. 991–994.

[25]Smith, R. S., and Dahleh, M. (eds.), *The Modeling of Uncertainty in Control Systems: Proceedings of the 1992 Santa Barbara Workshop*, Springer–Verlag, New York, 1994.

[26]Jabbari, F., and Gibson, J., "Adaptive Identification of a Flexible Structure by Lattice Filters," *Journal of Guidance, Control, and Dynamics*, Vol. 12, No. 4, 1989, pp. 548–554.

[27]Bayard, D. S., Yam, Y., and Mettler, E., "On the Integration of On-orbit System Identification with Modern Robust Control Tuning," *Proceedings of USAF/NASA Conference on System Identification and Health Monitoring of Precision Space Structures*, California Inst. of Technology, Pasadena, CA, 1990, pp. 1–22.

[28]Karlov, V., Miller, D., Vander Velde, W., and Crawley, E., "Identification of Model Parameters and Associated Uncertainties for Robust Control Design," *Journal of Guidance, Control, and Dynamics*, Vol. 17, No. 3, 1994, pp. 495–504.

[29]Yam, Y., Bayard, D., and Scheid, R., "Integrated Identification and Robust Control Tuning for Large Space Structures," *Proceedings of the American Control Conference*, 1990, pp. 2424–2429.

[30]Yam, Y., Bayard, D. S., and Scheid, R., "Frequency Domain Identification for Robust Large Space Structures Control Design," *Proceedings of the American Control Conference*, June 1991, pp. 3021–3023.

[31]Bayard, D., Yam, Y., and Mettler, E., "A Criterion for Joint Optimization of Identification and Robust Control," *IEEE Transactions on Automatic Control*, Vol. 37, July 1992, pp. 986–991.

[32]Phan, M., Juang, J.-N., Horta, L., and Longman, R., "System Identification from Closed-Loop Data with Known Output Feedback Dynamics," *Journal of Guidance, Control, and Dynamics*, Vol. 17, No. 4, 1994, pp. 661–669.

[33]Juang, J.-N., and Phan, M., "Identification of System, Observer, and Controller from Closed-Loop Experimental Data," *Journal of Guidance, Control, and Dynamics*, Vol. 17, No. 1, 1994, pp. 91–96.

[34]Van den Hof, P. M., and Schrama, R. J., "An Indirect Method for Transfer Function Estimation from Closed Loop Data," *Automatica*, Vol. 29, No. 6, 1993, pp. 1523–1527.

[35]Ljung, L., *System Identification, Theory for the User*, Information and System Sciences Ser., Prentice–Hall, Englewood Cliffs, NJ, 1987.

[36]Ljung, L., "Information Contents in Identification Data from Closed-Loop Operation," *Proceedings of the IEEE Control Decision Conference*, Inst. of Electrical and Electronics Engineers, New York, 1993, pp. 2248–2252.

[37]Eldred, D. B., and O'Neal, M. C., "The JPL Phase B Testbed Facility," *ADPA/AIAA/ASME/SPIE Conference on Active Materials and Adaptive Structures*, Nov. 1991.

[38]Spanos, J. T., and Kissil, A., "Modeling and Identification of the JPL Phase B Testbed," *ADPA/AIAA/ASME/SPIE Conference on Active Materials and Adaptive Structures*, Nov. 1991.

[39]Smith, R., Chu, C.-C., and Fanson, J. L., "The Design of H_∞ Controllers for an Experimental Non-Collocated Flexible Structure Problem," *IEEE Transaction Control System Technology*, Vol. 2, No. 2, 1994, pp. 101–109.